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A correction-to-scaling critical exponent for fluids at order ε^3

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Abstract. The critical exponent ω_5, which is a correction to scaling absent in the Ising model, corresponding to insertions of the operator ϕ^5 in a ϕ^4 theory near four dimensions, is calculated to third order in 4D.

One of the central themes in critical phenomena is universality, which states that certain critical properties of widely different substances are the same (Kadanoff 1971). In particular, it was expected that fluids, near their critical points, behave like an Ising model (Jasnow and Wortis 1968). Thus, for a number of years when experiments on fluids appeared to show non-Ising critical exponents (see e.g. Levelt-Sengers and Sengers 1975), some researchers (Valls and Hertz 1978) looked for a universality class for fluids distinct from the Ising one.

Recently, between the improvements of experimental techniques, with which reduced temperatures of 10^{-5} can be attained (for a comprehensive review, see Sengers (1981)), and the better theoretical understanding of corrections to scaling (Nicoll and Zia 1981, Nicoll 1981, Vause and Sak 1980), most would agree that fluids belong in the Ising universality class.

Even though both fluids and (uniaxial) magnets belong to the Ising class, there is one important difference: the symmetry associated with 'up' and 'down' in magnets is absent in fluids. There is no exact vapour–liquid symmetry. Amplitudes associated with the asymmetry are zero in the magnet. Thus, the corrections to scaling for fluids differ from those for magnets: more terms are expected for the former. In this note we report the results of the calculations to $O(\varepsilon^3)$ within the context of ϕ^4 theory in $d=4-\varepsilon$ dimensions.

The techniques for calculating such exponents are well documented (Wegner 1972, Amit 1978, Brézin *et al* 1976, Amit *et al* 1977). Our work is an extension of Nicoll and Zia (1981), where it was pointed out that $\int\phi^5$ and $\int\phi^2\nabla^2\phi$ are *the* two operators of major concern. Eigenoperators at $O(\varepsilon)$, they mix in higher order. The anomalous dimension in this basis is a non-diagonal matrix. To $O(\varepsilon^3)$, it is

$$\begin{pmatrix} -\frac{10}{3}\varepsilon+\frac{745}{324}\varepsilon^2-\frac{106\,715+103\,680\zeta}{34\,992}\varepsilon^3 & \frac{1}{18}\varepsilon^2+\frac{37}{1944}\varepsilon^3 \\ \frac{20}{3}\varepsilon-\frac{740}{81}\varepsilon^2+O(\varepsilon^3) & -\frac{4}{3}\varepsilon-\frac{95}{324}\varepsilon^2-\frac{2687-5184\zeta}{34\,992}\varepsilon^3 \end{pmatrix}$$

where $\zeta \equiv \zeta(3) \simeq 1.20$. The eigenvalues†, in the notation of Nicoll and Zia (1981), are

$$\lambda_5 = -\frac{10}{3}\varepsilon + \frac{685}{324}\varepsilon^2 - \frac{107\,855 + 103\,680\zeta}{34\,992}\varepsilon^3, \tag{1}$$

$$\lambda_3 = -\frac{4}{3}\varepsilon - \frac{35}{324}\varepsilon^2 - \frac{1547 - 5184\zeta}{34\,992}\varepsilon^3. \tag{2}$$

Note that λ_3 checks with that obtainable via the equation of motion (Brézin *et al* 1974, Nicoll 1981):

$$\lambda_3 = \varepsilon + \tfrac{1}{2}\eta - (2 - 1/\nu). \tag{3}$$

Together with the canonical dimension, the 'new' exponent is

$$\omega_5 = 1 + \frac{11}{6}\varepsilon - \frac{685}{324}\varepsilon^2 + \frac{107\,855 + 103\,680\zeta}{34\,992}\varepsilon^3 + \mathrm{O}(\varepsilon^4). \tag{4}$$

This exponent enters into singular parts of thermodynamic functions via $\xi^{-\omega_5}$ where ξ is the correlation length. Thus, a positive ω_5 corresponds to corrections to scaling with contributions that scale as $|T - T_c|^{\Delta_5}$ where $\Delta_5 = \omega_5\nu$.

Putting $\varepsilon = 1$ in (4) leads to the very large value of 7.36. This is hardly unexpected since it is well known that the asymptotic series in ε starts to diverge (often badly) at $\mathrm{O}(\varepsilon^3)$. The sequence of approximations for ω_5 at $\mathrm{O}(\varepsilon)$, $\mathrm{O}(\varepsilon^2)$ and $\mathrm{O}(\varepsilon^3)$ evaluated this way gives 2.83, 0.72 and 7.36 so far. This wild behaviour appears more tame if we look at the sequence after a (near-diagonal) Padé: 2.83, 1.85, 2.32. None of these results are conclusive except that they all support the estimate

$$\omega_5 \gtrsim 1.5 \qquad (\text{or } \Delta_5 \gtrsim 1.0). \tag{5}$$

If this estimate is to be trusted, a fit to experimental data probably does not need this contribution desparately, since this exponent would be twice as large as the smallest 'symmetric' correction-to-scaling exponent ω. From reliable large-order resummation estimates (Le Guillou and Zinn-Justin 1980), $\omega \simeq 0.78$ (or $\Delta \simeq 0.5$).

The present result should be viewed more as a stepping stone that contributes toward an eventual large-order resummation estimate rather than as an end to a calculation. Work is in progress to generalise the large-order resummation techniques to include the mixing of operators with different numbers of ϕ fields. A more reliable result for ω_5 would be very desirable in the construction of an asymmetric equation of state (Nicoll, unpublished) leading to the familiar asymmetric co-existence curve.

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† There is a misprint in formula (3.4*b*) for λ_3 in Nicoll and Zia (1981).

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